DIFFUSION-MIGRATION DESCRIPTION OF FINELY DISPERSED IMPURITY PROPAGATION IN A TURBULENT JET

A. A. Vinberg, L. I. Zaichik,
and V. A. Pershukov

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Computations of the scattering of a finely dispersed impurity in a submerged jet are performed on the basis of a model taking account of turbulent diffusion and turbulent migration together.

Up to now a sufficiently large quantity of publications has appeared that are devoted to computations of turbulent two-phase jet flows, [1-5], say. Two approaches are applied here to describe the mechanism of inertial particle interaction with the turbulent flow. The first approach is based on a modified Prandtl mixing-path theory [1, 2]. The second approach is related to the generalization of differential models for single-phase flow based on the equations for turbulent energy and other fluctuation characteristics, on disperse streams [3-5]. A simple model is represented in this paper to describe finely-dispersed impurity scattering within the framework of the second direction.

Particles can be provisionally separated according to the nature of the behavior in a turbulent stream into fine, whose dynamic relaxation time τ is less (or of the same order) than the characteristic lifetime of the power-intensive moles (the integral time scale of turbulence) T, and coarse whose relaxation time significantly exceeds the turbulence time scale. Many effects characteristic for coarse particles (in particular, the average slip with respect to the carrying current and the appearance of the Magnus force because of particle rotation) turn out to be secondary for fine particles and interaction with turbulent fluctuations turns out to be primary; forces related to the inhomogeneity of the velocity fluctuation distribution of the carrying phase and the impurity concentration, turbulent migration and turbulent diffusion, play the most substantial part. We henceforth limit ourselves to the examination of disperse flows containing fine particles in a small (and moreover, volume) concentration.

The mass and moment conservation equations of a discrete (solid) phase are written within the framework of the mechanics of interacting media [6] in the form

$$\frac{\partial \varphi}{\partial t}+\frac{\partial \varphi v_k}{\partial x_k}=0, \tag{1}$$

$$\frac{d\varphi v_i}{\partial t}+\frac{\partial \varphi v_i v_k}{\partial x_k}=\frac{\varphi (u_i-v_i)}{\tau}. \tag{2}$$

The right side of (2) describes the force of viscous interaction between the phases in a Stokes approximation. It is assumed here that the density of the carrying gas phase is substantially less than the density of the particle material and, consequently, the forces due to the pressure gradient, the apparent mass, and the nonstationarity of the flow (Bass force) cannot be taken into account.

Let us take the average of (1) and (2) over the ensemble of realizations of a turbulent flow in such a manner that the relationship $\langle \phi v_i' \rangle = 0$ would be satisfied. This method of taking the average of the solid phase characteristics by using the particle concentration as a weight function is analogous to the known method of taking the Faure average in the theory of single-phase flows with variable density [7]. The averaged equations (1) and (2) take the form

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$$\frac{\partial\Phi}{\partial t}+\frac{\partial\Phi V_k}{\partial x_k}=0, \tag{3}$$

$$\frac{\partial\Phi V_i}{\partial t}+\frac{\partial\Phi V_i V_k}{\partial x_k}=-\frac{\partial\Phi\langle v_i' v_k'\rangle}{\partial x_k}+\frac{\Phi(U_i-V_i)}{\tau}+\frac{\langle\varphi' u_i'\rangle}{\tau}, \tag{4}$$

where $\langle v_i' v_k'\rangle = \langle\varphi v_i' v_k'\rangle/\Phi$ is the turbulent stress tensor in the solid phase.

Let us note that taking the average of the gas phase characteristics is satisfied by the usual method without utilizing ϕ as a weight function.

We use the equation for disperse phase concentration fluctuations (1) obtained from (1) under the assumption that the term due to the gradient of the average concentration to calculate the correlation $\langle\phi' u_i'\rangle$ in (4)

$$\frac{\partial\varphi}{\partial t}=-v_k'\frac{\partial\Phi}{\partial x_k}.$$

Integrating this expression with respect to the time, then multiplying by $u_i'(t)$ and taking the average over the ensemble of turbulent realizations, we obtain the following gradient representation

$$\langle\varphi' u_i'\rangle=-\tau g\langle u_i' u_k'\rangle\frac{\partial\Phi}{\partial x_k},\quad g=\frac{1}{\tau}\int_0^\infty\left[1-\exp\left(-\frac{S}{\tau}\right)\right]F(S)\,dS, \tag{5}$$

where $F(S)=\langle u_i'(t)u_j'(t+S)\rangle/\langle u_i'(t)u_j'(t)\rangle$ is the correlation function of the gas velocity fluctuations along the particle trajectory.

Taking the relationship (5) into account (4) takes the form

$$\frac{\partial V_i}{\partial t}+V_k\frac{\partial V_i}{\partial x_k}=-\frac{\partial\langle v_i' v_k'\rangle}{\partial x_k}+\frac{U_i-V_i}{\tau}-\frac{D_{ik}}{\tau}\frac{\partial\ln\Phi}{\partial x_k}, \tag{6}$$

where $D_{ik}=\tau(\langle v_i' v_k'\rangle+g\langle u_i' u_k'\rangle)$ is the turbulent diffusion tensor of the particles. The first term in the right side of (6) describes the origination of turbulent stresses in the solid phase because of particle involvement in the fluctuating motion of the carrying stream and this latter governs the diffusion transfer of the momentum due to the concentration gradient.

The system of equations of motion of the solid phase (3) and (6) agree with the corresponding equations for the first two moments obtained in [8] from the equation for the particle distribution probability density in the phase space of the coordinates and velocities. These equations differ somewhat from the equations ordinarily utilized, for instance, [3-5], obtained by averaging without introducing the concentration as weight function. Although both approaches (averaging methods) are equivalent in principle, construction of relationships for the turbulent diffusion coefficient turns out to be more simple in this case.

The following diffusion equation for the particle concentrations is obtained from (3) and (6)

$$\tau\frac{\partial^2\Phi}{\partial t^2}+\frac{\partial\Phi}{\partial t}+\frac{\partial}{\partial x_k}V_k\Phi=\frac{\partial}{\partial x_i}\left[(D_{ik}+\tau V_i V_k)\frac{\partial\Phi}{\partial x_k}+\right.$$
$$\left.+\tau\Phi\frac{\partial}{\partial x_k}(\langle v_i' v_k'\rangle+V_i V_k)\right]. \tag{7}$$

It is seen that in contrast to the process of noninertial impurity scattering described by a parabolic type diffusion equation, the change in inertial particle concentration is described by a hyperbolic type diffusion equation. As $\tau\to 0$ (7) goes over into the usual diffusion equation for a noninertial impurity.

Turbulent stresses in the solid phase are expressed in terms of the Reynolds stress in the gas phase by using the known relationship [9]

$$\langle v_i' v_k'\rangle=f\langle u_i' u_k'\rangle,\quad f=\frac{1}{\tau}\int_0^\infty\exp\left(-\frac{S}{\tau}\right)F(S)\,dS. \tag{8}$$

It follows from (8) that the fine particles are involved completely in the turbulent motion of the carrying flow ($f\to 1$ as $\tau/T\to 0$), and, conversely coarse particles are not involved in the fluctuating motion ($f\to 0$ as $\tau/T\to\infty$). Taking (5) and (8) into account, the particle turbulent diffusion tensor is determined by the relationship $D_{ik} = T\langle u'_i u'_k \rangle$, i.e., agrees with the expression for a noninertial impurity (which corresponds to the Chen theorem [9]).

Approximating F(s) by a step function as in [10]

$$F(S) = \begin{cases} 1 \text{ for } 0 \leqslant S \leqslant T, \\ 0 \text{ for } S > T, \end{cases}$$

we obtain the following expression for the coefficients of particle involvement in the fluctuating motion of the carrying flow:

$$f = 1 - \exp(-1/\Omega),\ g = 1/\Omega - 1 + \exp(-1/\Omega).$$

The finely dispersed impurity ($\Omega \lesssim 10$) distribution in the flows, whose longitudinal scale of parameter variation is substantially greater than the transverse scale (flows in thin jets are among this type in particular), can be analyzed in a diffusion-migration approximation obtained from (7) when neglecting convective terms in the right side. For a stationary axisymmetric flow (7) takes the following form in a boundary layer theory approximation

$$\frac{\partial r U_x \Phi}{\partial x} + \frac{\partial r U_r \Phi}{\partial r} = \frac{\partial}{\partial r}\left[r\left(D_t \frac{\partial \Phi}{\partial r} + \Phi \frac{\partial q D_t}{\partial r}\right)\right], \quad (9)$$

where $D_t = T\langle u_r'^2 \rangle = \nu_t/\mathrm{Sc}_t$ is the turbulent diffusion coefficient.

The solution of (9) is a simpler problem as compared with the solution of (3) and (6) since the computation is performed in a single-velocity approximation in this case; however, the migration transport due to inhomogeneity of the turbulent fluctuations of the carrying flow is also taken into account in addition to diffusion transport characteristic for a noninertial impurity. As the particle inertia increases the coefficient of migration q grows from zero to one; consequently, if the scattering of very fine particles ($\Omega \ll 1$) is determined primarily by the turbulent diffusion process, then as the particle inertia grows the role of the migration mechanism is raised and the propagation with respect to the coarse particles ($\Omega \sim 1$) is determined to a substantial degree by the turbulent migration process. It should be noted that the possibility of explaining the effect of tying the finely-disperse impurity in the initial section by the influence of turbulent migration (turbophoresis) was mentioned earlier in [3].

Limiting ourselves to the consideration of disperse flows for a small particle mass concentration ($\rho_2\Phi_0/\rho_1 \ll 1$) we neglect the reverse influence of the particles on the average and fluctuation characteristics of the carrying medium. In this case a standard k–ε model of turbulence with the correction introduced in [11] in the equation for dissipation that takes account of the effect of vortex extension in a circular jet is used to describe the gas phase turbulent energy and velocity profiles in this case. The square of the velocity fluctuation in the radial direction is determined by the relationship $\langle u'^2 \rangle = 2K/3$ (satisfied certainly for jet flows [1]) while the turbulence time scale is $T = \alpha K/\varepsilon$, where $\alpha = 0.17$ (which corresponds to $\mathrm{Sc}_t = 0.8$ with $\nu_t = c_\mu K^2/\varepsilon$, $c_\mu = 0.09$ taken into account).

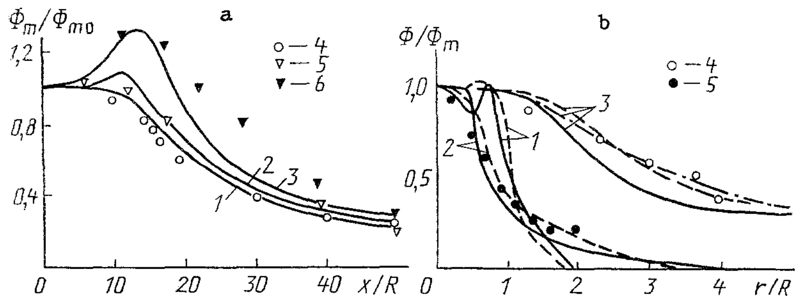

Fig. 1. Impurity concentration distribution along the jet axis (a) and along the jet section (b); a) 1, 4) d = 0; 2, 5) 7 μm; 4) [12]; 5, 6) [14]; b) d = 0 (dash-dot line), d = 7 μm (solid line), d = 32 μm (dashes); 1) x/R = 4; 2) x/R = 12; 3) x/R = 40; 4) [1]; 5) [13].

The comparison between the computed and experimental data on the change in the impurity concentration along the jet axis is shown in the Fig. 1a for different values of the particle radius. In the case of a noninertial impurity (d = 0) a monotonic diminution in the concentration is observed as the distance from the nozzle exit increases. Upon inserting a finely-dispersed inertial impurity in the flow in the initial section, growth of its concentration (the so-called tying effect) occurs, and then the particle concentration, as for a noninertial impurity, diminishes according to the law $\Phi \sim 1/x$. The increase in the impurity concentration is explained by the nonmonotonic nature of the turbulent energy change in the axial and radial directions for a high turbulence level in the jet initial section. This circumstance indicates the substantial influence of particle interaction with the turbulent fluctuations of the carrying flow on the nature of impurity scattering. As is seen from Fig. 1, the clearly defined concentration maximum is observed as the particle size grows, which is explained by the rise in the role of the turbulent migration. Therefore, the tying effect, consisting of a nonmonotonic change in the particle concentration that is detected in experiments, is reproduced sufficiently well for a finely-dispersed impurity within the framework of the approximation under consideration because of taking tubulent migration into account. A certain discrepancy between the computed and experimental data is apparently explained by neglecting the reverse influence of particles on the average and fluctuating flow configuration.

Graphs of the change in impurity concentration in the jet transverse section are presented in Fig. 1b at different distances from the nozzle exit. It is seen that the change in concentration over the section is nonmonotonic in nature in the jet the initial section as in the axial direction. This effect is determined by the presence of a quite definite maximum shifted from the jet axis, in the actual turbulent energy distribution obtained both in computations and in experiments [15]. As the distance from the nozzle exit increases, the value of the maximum in the turbulent energy profile diminishes and the transverse particle concentration distribution becomes monotonic. In the particle size range under investigation the radial concentration distribution profile is filled up more as compared with the axial velocity profile; this fact indicates that the effective Shmidt number is greater than one. This circumstance is stressed repeatedly in investigations devoted to two-phase jet flows ([1], for instance). However, in contrast to [1], the effect mentioned is taken into account in the present model not by increasing the value of Sc_t but because of taking turbulent migration into account. It should be noted that at large distances from the nozzle exit ($x/R > 40$) the inertial impurity distributions differ slightly from the noninertial, as is explained by the low level and smooth distribution of the turbulent energy, and therefore, the insignificant role of the turbulent migration.

As a whole, the diffusion-migration model permits satisfactory description of the finely-dispersed impurity distribution in both the longitudinal and transverse directions.

NOTATION

u_i, U_i, v_i, V_i are the actual and averaged gas and solid phase velocities; φ, Φ is the actual and averaged volume impurity concentration; ρ_1, ρ_2 are the gas and particle densities; d is the particle diameter; $\tau = \rho_2 d^2/18\rho_1\nu$ is the particle relaxation time; R is the nozzle radius; ν_t is the gas coefficient of turbulent viscosity; $Sc_t = \nu_t/D_t$ is the turbulent Schmidt number; $k = \langle u_k' u_k' \rangle/2$ is the turbulent energy; ε is the turbulent energy dissipation; x, r are coordinates in the axial and radial directions; $\Omega = \tau/T$ is the particle inertia parameter; $q = \Omega f$ is the migration coefficient. Subscripts: m are values on the axis, and 0 are values on the nozzle exit.

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HEAT-TRANSFER INTENSITY FROM SWIRLING DISPERSE FLOW TO CYCLONE-CHAMBER WALL

V. A. Kirakosyan, A. P. Baskakov,
E. Yu. Lavrovskaya, and Yu. A. Popov

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On the basis of a two-layer scheme of turbulent motion around a wall, an expression is obtained for the heat-transfer coefficient from swirling disperse flow to the cyclone wall.

In engineering practice, the heat extraction from the lateral surface of the cyclone must often be estimated in calculating designing a cyclone-type heat exchanger. Accurate calculation of the heat fluxes from swirling disperse flow to the cyclone wall requires combined solution of the equations of energy and motion of the gas and the particles. Analytical solution of this problem is not possible. Usually, various empirical dependences which are only valid for the values of the cyclone structural parameters corresponding to the particular experiment [1-4] are used to calculate the convective heat-transfer coefficient from the gases to the lateral surface of the cyclone. In addition the empirical dependences proposed in the literature take no account of the influence of dust content on the heat-transfer coefficient. It is expedient to generalize these experimental data using the well-known methods of approximate solution of analogous problems — in particular, the two-layer scheme of turbulent motion around a wall [5]. Following this method, it is first assumed that, on reaching the cyclone, the gas and particles move downward to the wall region of the apparatus (as indicated by experiments) and are of identical temperature.

By analogy with the problem of flow around a plane plate, it is assumed that the heat flux and tangential stress remain constant over the boundary layer and are equal to the corresponding values at the cyclone surface ($q = q_{wa}$, $\tau = \tau_{wa}$). Then the following expressions may be written

$$\frac{\tau_{wa}}{\rho} = -(\nu + \nu_{T}) \frac{1}{r} \frac{\partial (Vr)}{\partial r}, \quad (1)$$

$$\frac{q_{wa}}{\rho} = -C_{ef}(a + a_{T}) \frac{\partial T}{\partial r}, \quad (2)$$

where $C_{ef} = C_g(1 + \mu C_s/C_g)$ is the effective specific heat of the disperse flow (it is assumed that, on account of the high intensity of interphase heat transfer, the temperature of the gas and the particles is the same).

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